Dynamical response functions in correlated fermionic systems

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Abstract

Response functions in nuclear matter at finite temperature are considered beyond the usual Hartree-Fock (HF) plus Random Phase Approximation (RPA) scheme. The contributions due to the propagator for the dressed nucleons and the corresponding vertex corrections are treated in a consistent way. For that purpose a semi-realistic Hamiltonian is developed with parameters adjusted to reproduce the nucleon self-energy as derived from realistic nucleon-nucleon interactions. For a scalar residual interaction the resulting response functions are very close to the RPA response functions. However, the collective modes, if present, get an additional width due to the coupling to multi-pair configurations. For isospin dependent residual interactions we find strong modifications of isospin response functions due to multi-pair contributions in the response function. Such a modification can lead to the disappearance of collective spin or isospin modes in a correlated system and shall have an effect on the absorption rate of neutrinos in nuclear matter.

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I. INTRODUCTION

The shell-model or independent particle model (IPM) has been very successful in describing basic features of nuclear systems. This means that nuclei are considered as a system of nucleons moving independently in a mean field and the residual interaction between these particle or quasiparticles is supposed to be weak. Therefore the response of the the system to an external perturbation can be calculated within the Fermi Liquid theory [1] in terms of linear response functions. These response functions are calculated assuming a Hartree-Fock (HF) propagator for the particle-hole excitations of the nucleons and including the residual interaction by means of the Random Phase Approximation (RPA) approximation. In the long-wavelength limit or external perturbations with low momentum transfer, the residual interaction between the quasiparticles is usually parameterized in terms of Landau parameters.

This HF plus RPA scheme is typically used to determine e.g. the neutrino propagator in hot and dense nuclear matter [2, 3, 4, 5, 6, 7] and it has been found that the neutrino opacity is very sensitive to the details of these response functions. This quantity is very crucial for the simulation of astrophysical objects like the explosion of supernovae or the cooling mechanism for neutron stars [8, 9].

The study of the response is also very important to determine the propagator of mesons or a photon in the nuclear medium. Therefore such investigations have to be performed to explore e.g. the possibility for a pion condensation[10, 11] or the production and emission of mesons and photons from the hot dense matter obtained in heavy ion reactions [12, 13, 14, 15, 16, 17]. Last not least the response function is also reflecting the excitation modes of nuclei.

However, the simple HF plus RPA scheme outlined above is applicable to nuclear systems only if effective nucleon-nucleon (NN) interactions like Skyrme[18] or Gogny[19] forces are employed. The IPM fails completely if realistic NN interactions are considered, which have been adjusted to describe the NN scattering data. Trying to evaluate the energy of nuclear matter from such realistic interaction within the HF approximation yields positive energies, i.e. unbound nuclei[20]. The reason of this deficiency of the HF approximation in nuclear physics are the correlations beyond the IPM approach, which are induced from the strong short-range and tensor components of a realistic NN interaction.

These correlations have a significant effect on the single-particle propagator for a nucleon in the nuclear medium. The spectral function still exhibits a quasiparticle peak. A sizable fraction, however, of the strength occurs at energies above and below the quasiparticle peak. For hole-states one typically observes that around 15 % of the spectral strength is shifted to energies above the Fermi energy [21, 22, 23] which means that the occupation probability of those states is reduced from 100 % in the case of the IPM approach to around 85 %. Another fraction of the hole-strength is shifted to energies below the quasiparticle energies, which means that it should be found in nucleon knock-out experiments at large missing energies. These effects of correlations on the spectral distribution are confirmed in (e, e'p) experiments (see e.g.[24]).

In lowest order this redistribution of the strength in the single-particle spectral function is due to the admixture of two-hole one-particle and two-particle one-hole contributions to the propagator in the HF field. Therefore one may feel the temptation to use these correlated propagators and evaluate the nuclear response function in terms of these dressed propagators. In this way one is including two-particle two-hole admixtures to the particle-hole response function. As we can expect from the discussion above and as we will see below, such a procedure leads to response functions which, comparing to the HF plus RPA response, exhibit a significant shift of the excitation strength to larger energies.

We will also see, however, that such a significant shift of the excitation strength, can in general not be consistent with the energy weighted sum-rules, which are observed in the HF plus RPA scheme. It is well known that it is rather difficult to develop a symmetry conserving approach for the evaluation of Green's functions which accounts for correlations beyond the HF plus RPA approximation. In the case of the response function this requires a consistent treatment of propagator and vertex corrections. In this manuscript we will follow the general recipe of Baym and Kadanoff[25, 26] for calculating the in-medium coupling of an external perturbation to dressed nucleons in a self-consistent way.

This procedure leads to an integral equation, a Bethe-Salpeter equation for the dressed vertex. The in-medium vertex has the structure of a three-point Green's functions. For dressed (off-shell) nucleons it is a function of two momenta and two energies. Since such calculations are rather involved, only a few exist for the density response function [27, 28, 29]. In order to make these calculation feasible, we define a simple interaction

model. The nucleons are dressed by a mean-field and a residual interaction. The residual interaction is taken selfconsistently to the second order. The parameters of the interaction are adjusted to reproduce the main features of single-particle spectral functions derived from realistic NN interactions.

After this introduction we present in section 2 our interaction model and the adjustment of its parameters to reproduce the nucleon self-energy derived from a realistic interaction. The evaluation of the response functions with a consistent treatment of propagator and vertex corrections is outlined in section 3. Numerical results for symmetric nuclear matter and pure neutron matter are presented in section 4. There we also discuss the effect of multi-pair contributions to the response functions at high excitation energies and its relation to the spin-isospin structure of the residual interaction and the consequences for the damping of collective modes. The final section summarizes the main conclusions of this study.

II. MEAN-FIELD AND RESIDUAL INTERACTION

Calculations of the response function in nuclear matter are usually restricted to the HF plus RPA approximation, employing parameterizations of the effective NN interaction. There exist many relatively simple and successful parameterization of the mean-field Hamiltonian for nuclear systems, e.g. The Skyrme interaction [18] and the Gogny interaction [19]. Usually such effective interactions include spin and isospin dependent terms, and also density dependent terms. The Skyrme interaction is a zero range interaction with velocity dependent terms, for which a complete calculation of the RPA response is possible [30]. In general, however, the calculation of the RPA response requires the consideration of a nontrivial sum of exchange terms[10, 31], which are often approximated. Usually the response function for finite range interactions is calculated expanding the interaction in Landau parameters [32].

In this study we want to analyze the linear response functions in a fermionic systems when the correlations of the system that we take into account go beyond the mean-field approximation. We suppose that the interactions between the nucleons are given by a mean-field potential and a residual interaction. The mean-field interaction that we take



FIG. 1: Diagrams for the self-energy. The first two diagrams are the Hartree-Fock contribution for the Gogny interaction (the dashed line). The last diagram is the contribution of the residual interaction in the second order.

is based on the Gogny parameterization

$$V_{mf}(1,2) = \sum_{i} (W_i + B_i P^{\sigma} - H_i P \tau - M_i P^{\sigma} P^{\tau}) e^{-(\mathbf{r_1} - \mathbf{r_2})^2/\mu_i^2} + \sum_{j} t_3^j (1 + x_3^j P^{\sigma}) \rho^{\sigma_j} \delta^3(\mathbf{r_1} - \mathbf{r_2}) .$$
(1)

The first term is a sum of two Gaussians giving a finite range interaction and the second term is a sum of two zero-range density dependent interactions [33], $P^{\sigma} = \frac{1}{2}(1 + \sigma_1\sigma_2)$ and $P^{\tau} = \frac{1}{2}(1 + \tau_1\tau_2)$. The residual interaction is taken in a very simple form [34]

$$V_{res}(\mathbf{r_1} - \mathbf{r_2}) = V_0 e^{-(\mathbf{r_1} - \mathbf{r_2})^2 / 2\eta^2}$$
, (2)

with the parameters $V_0 = 453 \text{MeV}$, $\eta = 0.57 \text{fm}$. The single particle propagator is calculated by taking the mean field contributions only for the Gogny interaction (1) and the second order direct Born term for the residual interaction (2). The relevant diagrams are shown in Fig. 1.

The residual interaction induces a finite width to the nucleon excitations in the medium. Such a dressing of nucleons is expected in any approach going beyond the simple mean-field. Calculation for the system including the mean-field and residual interactions are performed in the real-time representation for the thermal Green's functions [35]. The iterated system of equations includes expressions for the self-energies,

$$\Sigma_{mf}(p) = V_{mf}(0)\rho - Tr \int \frac{d^3k}{(2\pi)^3} P^{\sigma} P^{\tau} V_{mf}(\mathbf{p} - \mathbf{k}) n(k) ,$$

$$\Sigma^{>(<)}(\mathbf{p}, \omega) = 4i \int \frac{d^3p_1 d\omega_1 d^3p_2 d\omega_2}{(2\pi)^8} V_{res}^2(\mathbf{p} - \mathbf{p_1}) G^{>(<)}(\mathbf{p_1}, \omega_1) G^{<(>)}(\mathbf{p_2}, \omega_2)$$

$$G^{>(<)}(\mathbf{p} - \mathbf{p_1} + \mathbf{p_2}, \omega - \omega_1 + \omega_2) ,$$
(3)

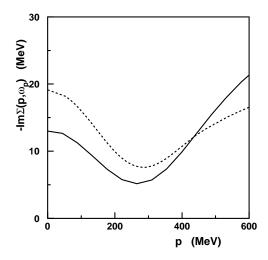


FIG. 2: Imaginary part of the self-energy at the quasi-particle pole from the residual interaction (solid line) and from a self-consistent T-matrix calculation [36] (dashed line). The curves shown correspond to symmetric nuclear matter at normal nuclear density and temperature $T=15 \, MeV$.

$$\Sigma^{r(a)}(\mathbf{p},\omega) = \Sigma_{mf}(p) + \int \frac{d\omega_1}{2\pi} \frac{\Sigma^{<}(\mathbf{p},\omega_1) - \Sigma^{>}(\mathbf{p},\omega_1)}{\omega - \omega_1 \pm i\epsilon} , \qquad (4)$$

and the Dyson equation for the retarded (advanced) Green's functions

$$G^{r(a)}(\mathbf{p},\omega) = \frac{1}{\omega - \mathbf{p}^2 / 2m - \Sigma^{r(a)}(\mathbf{p},\omega)}.$$
 (5)

The Green's functions

$$G^{>}(\mathbf{p},\omega) = -i(1 - f(\omega)) A(\mathbf{p},\omega) ,$$

$$G^{<}(\mathbf{p},\omega) = if(\omega) A(\mathbf{p},\omega)$$
(6)

are written using the Fermi distribution $f(\omega)$ and the spectral function

$$A(\mathbf{p},\omega) = -2\operatorname{Im}G^{r}(\mathbf{p},\omega) . \tag{7}$$

The nucleon momentum distribution is

$$n(p) = \int \frac{d\omega}{2\pi} A(p,\omega) f(\omega)$$
 (8)

and the chemical potential is adjusted at each iteration to reproduce the assumed density

$$\rho = 4 \int \frac{d^3 p}{(2\pi)^3} n(p) \ . \tag{9}$$

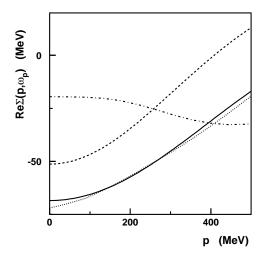


FIG. 3: Real part of the self-energy at the quasi-particle pole from the residual interaction (dashed-dotted line) and from the modified mean-field interaction (dashed line). The sum of the two contributions is shown as the solid line and compared the original Gogny potential (dotted line).

The self-consistent equations for the one-body properties have been solved within similar approximations by several groups [13, 14, 34, 37, 38]. The width obtained from a self-consistent calculation in the second order of the residual interaction is similar to the result obtained from a self-consistent T-matrix calculation using realistic bare nucleon-nucleon interaction (Fig. 2).

The real part of the self-energy Re $\Sigma^r(p,\omega)$ is the sum of the mean-field (Gogny) contribution and a dispersive one obtained from the dispersion relation in Eq. (4). This means that the real part of the self-energy at the quasiparticle pole $\omega_p = p^2/2m + \Sigma(p,\omega_p)$ is different from the Gogny single-particle potential. Accordingly we have modified some parameters of the Gogny interaction in order to have the same Fermi energy as function of density and a similar effective mass. In Fig. 3 we show the real part of the self-energy at the quasi-particle pole and compare it to the single-particle potential derived from the original Gogny interaction. This real part of the self-energy is the sum of the dispersive part and the mean-field contribution originating from the modified Gogny interaction, which are also shown.

parameter	Gogny D1P	Modif. I	Modif II
$\mu_1 \text{ (fm)}$	0.9	-	-
$\mu_2 \text{ (fm)}$	1.44	-	-
$W_1 \text{ (MeV)}$	-372.9	-	-
$W_2 \text{ (MeV)}$	34.6	-	-
$B_1 \text{ (MeV)}$	62.7	-	-
$B_2 \text{ (MeV)}$	-14.1	-	-
$H_1 \text{ (MeV)}$	-464.5	-	-
$H_2 \text{ (MeV)}$	-70.9	-	-
$M_1 \text{ (MeV)}$	-31.5	38.5	38.5
$M_2 \; ({ m MeV})$	-21	-51	-51
σ_1	.33	-	-
σ_2	.92	-	-
$t_3^1 \text{ (MeV fm}^{3(\sigma_1+1)})$	1025.9	454.7	-245.3
$t_3^2 \; (\text{MeV fm}^{3(\sigma_2+1)})$	1025.9	-	-
$t_3^1 x_3^1 \text{ (MeV fm}^{3(\sigma_1+1)})$	1190	-	-
$t_3^2 x_3^2 \text{ (MeV fm}^{3(\sigma_2+1)})$	256	-	-
$t_3^0 \; ({ m MeV \; fm^3})$	0	478.4	803.4

TABLE I: Table of the parameters for the mean-field interaction. The first column corresponds to the Gogny D1P parameterization [33], the second and third columns are modifications of the mean-field interaction used in symmetric nuclear and neutron matter, respectively. A dash is put whenever the value of the corresponding parameter is not changed.

The parameters of the mean-field interaction are given in Table I. We modify the parameters M_i to reproduce the momentum dependence of the self-energy and also the density dependent zero range term, an additional term of the form $t_3^0 \delta^3(\mathbf{r}_1 - \mathbf{r}_2)$ is added to the mean-field interaction.

FIG. 4: The Bethe-Salpeter equation for the dressed vertex. The particle-hole irreducible kernel K is denoted by the box and the fat and the small dots denote the dressed and the bare vertices for the coupling of the external field to the nucleon. All the fermionic propagators are dressed by the self-energy as displayed in Fig. 1.

III. CORRELATIONS AND RESPONSE FUNCTIONS

In this section we discuss the linear response of a correlated system to an external perturbation. As it has already been mentioned above the evaluation of the RPA response function requires for a general interaction a solution of a Bethe-Salpeter (BS) equation with a non-trivial kernel which is due to the exchange terms in the NN interaction. Therefore one often simplifies the solution of the BS equation by a parameterization of the particle-hole interaction in terms of Landau parameters. E.g. the density response function is written using the zero order Landau parameter f_0 as

$$\Pi^{r}(p,\omega) = \frac{\Pi_{0}^{r}(p,\omega)}{1 - f_{0}\Pi_{0}^{r}(p,\omega)} , \qquad (10)$$

where $\Pi_0^r(p,\omega)$ is the response function of the free Fermi gas using an effective mass to describe the momentum dependence of the mean field.

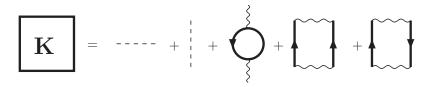
When the description of the correlated systems goes beyond the mean-field approximation the difficulty involved in a consistent calculation of the response function is severely increased. A naive calculation of the polarization bubble using dressed propagators

$$\Pi^{<(>)}(\mathbf{q},\Omega) = -4i \int \frac{d^3 p d\omega}{(2\pi)^4} G^{<(>)}(\mathbf{q} + \mathbf{p},\omega + \Omega) G^{>(<)}(\mathbf{p},\omega)$$
(11)

and

$$\Pi^{r(a)}(p,\omega) = \int \frac{d\omega_1}{2\pi} \frac{\Pi^{<}(p,\omega_1) - \Pi^{>}(p,\omega_1)}{\omega - \omega_1 \pm i\epsilon} , \qquad (12)$$

can be a rather poor estimate for the response function [29]. In particular, it severely violates the ω -sum rule for finite momentum q. This violation of the sum rule by the



$$=$$
 ---- + $\left| \begin{array}{c} + \\ \end{array} \right|$ $\left| \mathbf{K_{res}} \right|$

FIG. 5: The kernel of the Bethe-Salpeter equation corresponding to the self-energy in Fig. 1 containing a mean field and a residual interaction.

naive one-loop response function was recently noticed by Tamm et al. [39] in reply to an evaluation of a one-loop response function in terms of dressed propagators for the electrogas in metals and semiconductors[40].

For self-consistent approximation schemes a general recipe for calculating the inmedium coupling of the external potential to dressed nucleons is known [25, 26]. The in-medium vertex describing the coupling of the external perturbation to the nucleons nucleons is given by the solution of the BS equation (Fig. 4), where K denotes the particle-hole irreducible kernel. The kernel K of the BS equation should be taken consistently with the chosen expression for the self-energy. It is given by the functional derivative of the self-energy with respect to the dressed Green's function [25, 26] $K = \delta \Sigma / \delta G$.

The resulting kernel of the BS equation contains the usual mean-field interaction (direct and exchange terms, see first and second term in the representation of K displayed in Fig. 5) and additional diagrams which are due to the contributions of the residual interaction terms in the self-energy and are collected as K_{res} in Fig. 5.

Using the dressed vertex obtained as a solution of the BS equation the response function in the correlated medium can be obtained from the diagram in Fig. 6. Only one vertex in the loop includes in-medium modifications in order to avoid double counting.

The three-point Green's function for the coupling of the nucleon to an external field

FIG. 6: The polarization function expressed using the dressed vertex for the coupling of the external current to the dressed nucleon.



$$\Pi_{(ST)}$$
 $=$ $+$ $+$

FIG. 7: The Bethe-Salpeter equation with contributions only from the residual interaction (Eqs. 20, 21, and 23)

with momentum \mathbf{q} and energy Ω is

$$G_{(ST)}(x_1, t_1; x_2, t_2; \mathbf{q}, \Omega) = -\int d^3x \, dt \, \exp(-i\mathbf{q}\mathbf{x} + i\Omega t) \langle \mathcal{T}\Psi(x_1, t_1)\rho^{(ST)}(x, t)\Psi^{\dagger}(x_2, t_2) \rangle ,$$
(13)

where Ψ^{\dagger} , Ψ are the field creation and annihilation operators, $\rho^{(ST)}(x,t) = \Psi^{\dagger}(x,t)\Gamma^{0}_{(ST)}\Psi(x,t)$ denotes the bare coupling to the external field with the spin-isospin operators $\Gamma^{0}_{(ST)} = 1$, σ_{3} , τ_{3} , $\tau_{3}\sigma_{3}$ for the response functions denoted by spin S and isospin T equal to ST = 00, 10, 01, 11, respectively. The operator \mathcal{T} in this equation is the usual operator for the time ordering on the real time contour [35].

This three-point Green's function for the dressed coupling of the external field to the fermions in medium has a complicated analytical structure and depends on the incoming momentum \mathbf{q} and energy Ω [41]. Depending on the ordering of the times of the fermion operators one can define the smaller (larger) Green's functions $G_{(ST)}^{<(>)}(x_1, t_1; x_2, t_2; \mathbf{q}, \Omega)$ and also the retarded or advanced ones. In the momentum representation the three-point

Green's function depends on the momentum \mathbf{p} and energy ω of the incoming fermion and the momentum is $\mathbf{q} + \mathbf{p}$ and energy $\omega + \Omega$ of the outgoing fermion. We write the smaller (larger) Green's functions

$$G_{(ST)}^{<(>)}(\mathbf{q} + \mathbf{p}, \omega + \Omega; \mathbf{p}, \omega)$$
 (14)

and denote the retarded (advanced) Green's functions by

$$G_{(ST)}^{r(a)}(\mathbf{q} + \mathbf{p}, \omega + \Omega; \mathbf{p}, \omega)$$
 (15)

The response function can be expressed using this three-point Green's function (Fig. 6)

$$\Pi_{(ST)}^{r}(\mathbf{q},\Omega) = -iTr \int \frac{d^{3}pd\omega}{(2\pi)^{4}} \Gamma_{(ST)}^{0} G_{(ST)}^{<}(\mathbf{p} + \mathbf{q}, \omega + \Omega; \mathbf{p}, \omega) .$$
 (16)

The three-point Green's functions $G_{(ST)}$ can be written in terms of the in-medium (dressed) vertex $\Gamma_{(ST)}$ describing the in-medium coupling to the external perturbation

$$G_{(ST)}^{r(a)}(\mathbf{q} + \mathbf{p}, \omega + \Omega; \mathbf{p}, \omega) = G^{r(a)}(\mathbf{q} + \mathbf{p}, \omega + \Omega)\Gamma_{(ST)}^{r(a)}(\mathbf{q} + \mathbf{p}, \omega + \Omega; \mathbf{p}, \omega)G^{r(a)}(\mathbf{p}, \omega)$$
(17)

and

$$G_{(ST)}^{<(\gt)}(\mathbf{q} + \mathbf{p}, \omega + \Omega; \mathbf{p}, \omega) =$$

$$G^{r}(\mathbf{q} + \mathbf{p}, \omega + \Omega)\Gamma_{(ST)}^{r}(\mathbf{q} + \mathbf{p}, \omega + \Omega; \mathbf{p}, \omega)G^{<(\gt)}(\mathbf{p}, \omega)$$

$$+G^{r}(\mathbf{q} + \mathbf{p}, \omega + \Omega)\Gamma_{(ST)}^{<(\gt)}(\mathbf{q} + \mathbf{p}, \omega + \Omega; \mathbf{p}, \omega)G^{a}(\mathbf{p}, \omega)$$

$$+G^{<(\gt)}(\mathbf{q} + \mathbf{p}, \omega + \Omega)\Gamma_{(ST)}^{a}(\mathbf{q} + \mathbf{p}, \omega + \Omega; \mathbf{p}, \omega)G^{a}(\mathbf{p}, \omega) . \quad (18)$$

The dressed vertex $\Gamma_{(ST)}$ for the coupling of the external field to the nucleon is the solution of the Bethe-Salpeter equation displayed in Fig. 4.

As mentioned before the kernel of this BS equation has contributions from the mean-field and from the residual interaction. In this work we are interested in the role of the correlations going beyond the mean-field, which are described by means of the residual interaction discussed above. Therefore we will also consider the effects which are due to the mean field and the residual interactions in the kernel of the Bethe-Salpeter in separate steps. In a first step we will concentrate on the effects of the residual interaction and determine a response function $\Pi_{(ST)}$ res which includes vertex correction from the residual interaction (K_{res} in Fig. 5), only. The way to evaluate $\Pi_{(ST)}$ res will be discussed below.

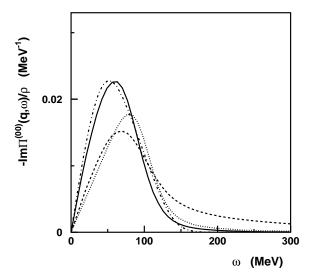


FIG. 8: The imaginary part of the response function as function of the energy for the momentum $q=220~{\rm MeV}$, at normal nuclear density and temperature 15 MeV, without including the RPA modifications of the response. The dashed-dotted line is the response function for the free Fermi gas with the same effective mass for the nucleons as in the correlated matter. The solid line denotes the response function including the dressing of the propagators and the vertex corrections (the density response). The dashed line is the naive one-loop response function with dressed propagators, but without vertex corrections. The dotted line is the response including the dressing of the propagators and the vertex corrections from the third diagram on the right hand side in Fig. 5 (spin-isospin channels 01, 10, and 11).

The modifications of the response function which are due to the mean-field interaction can be taken into account by a solution of a separate BS equation with a kernel including only this mean-field interaction and the zero order vertex being the vertex dressed by the residual interaction. In the following we approximate the solution of this second BS equation with the mean-field kernel using Landau parameters for the mean-field interaction. E.g. for the density response function we have

$$\Pi_{(00)}^{r}(p,\omega) = \frac{\Pi_{(00)\ res}^{r}(p,\omega)}{1 - f_0 \Pi_{(00)\ res}^{r}(p,\omega)}$$
(19)

and analogously for other channels. The Landau parameters are calculated using the modified Gogny mean-field with parameters from Table I. There is little change of these pa-

rameters when comparing with the Landau parameters obtained from the original Gogny interaction. Only in the scalar channel the interaction gets less attractive.

So we now turn to the effects of the residual interaction in the response function. In the real-time representation the Bethe-Salpeter equations including only the residual interaction in the kernel take the form

$$\Gamma_{(ST)}^{<(\gt)}(\mathbf{p} + \mathbf{q}, \omega + \Omega; \mathbf{p}, \omega) = iTr \int \frac{d^3 p_1 d\omega_1}{(2\pi)^4} \left(V^2(\mathbf{p} - \mathbf{p_1}) \Pi^{<(\gt)}(\mathbf{p} - \mathbf{p_1}, \omega - \omega_1) G_{(ST)}^{<(\gt)}(\mathbf{p_1} + \mathbf{q}, \omega_1 + \Omega; \mathbf{p_1}, \omega_1) + V(\mathbf{p_1}) V(\mathbf{p_1} + \mathbf{q}) \widetilde{\Pi}_{(ST)}^{<(\gt)}(\mathbf{p_1} + \mathbf{q}, \omega_1 + \Omega; \mathbf{p_1}, \omega_1) G^{<(\gt)}(\mathbf{p} - \mathbf{p_1}, \omega - \omega_1) \right)$$
(20)

and

$$\Gamma_{(ST)}^{r(a)}(\mathbf{p} + \mathbf{q}, \omega + \Omega; \mathbf{p}, \omega) = 1 + iTr \int \frac{d^3 p_1 d\omega_1}{(2\pi)^4} \Big($$

$$V^2(\mathbf{p} - \mathbf{p_1})\Pi^{<}(\mathbf{p} - \mathbf{p_1}, \omega - \omega_1)G_{(ST)}^{r(a)}(\mathbf{p_1} + \mathbf{q}, \omega_1 + \Omega; \mathbf{p_1}, \omega_1)$$

$$+V(\mathbf{p_1})V(\mathbf{p_1} + \mathbf{q})\widetilde{\Pi}_{(ST)}^{<}(\mathbf{p_1} + \mathbf{q}, \omega_1 + \Omega; \mathbf{p_1}, \omega_1)G^{r(a)}(\mathbf{p} - \mathbf{p_1}, \omega - \omega_1)$$

$$+V^2(\mathbf{p} - \mathbf{p_1})\Pi^{r(a)}(\mathbf{p} - \mathbf{p_1}, \omega - \omega_1)G_{(ST)}^{>}(\mathbf{p_1} + \mathbf{q}, \omega_1 + \Omega; \mathbf{p_1}, \omega_1)$$

$$+V(\mathbf{p_1})V(\mathbf{p_1} + \mathbf{q})\widetilde{\Pi}_{(ST)}^{r(a)}(\mathbf{p_1} + \mathbf{q}, \omega_1 + \Omega; \mathbf{p_1}, \omega_1)G^{>}(\mathbf{p} - \mathbf{p_1}, \omega - \omega_1)\Big). (21)$$

The terms in these equations containing the one-loop polarization function $\Pi^{<(>)}$ or $\Pi^{r(a)}$ of Eq.(11) and Eq.(12), respectively correspond to the second diagram on the right-hand side of the first line in Fig. 7. This is the lowest order contribution of the so-called induced interaction [11, 42, 43, 44] to the response function and therefore we will refer to it as the induced interaction term in the discussion below. The other terms refer to vertex corrections, which represented by the third diagram on the right-hand side of the first line in Fig. 23. They contain a three-point function which is displayed in the second line of Fig. 7 and is defined by

$$\widetilde{\Pi}_{(ST)}^{<(>)}(\mathbf{q} + \mathbf{p}, \omega + \Omega; \mathbf{p}, \omega) = -iTr \int \frac{d^3 p_1 d\omega_1}{(2\pi)^4} \Big(G_{(ST)}^{<(>)}(\mathbf{q} + \mathbf{p} + \mathbf{p_1}, \omega + \Omega + \omega_1; \mathbf{p} + \mathbf{p_1}, \omega + \omega_1) G^{>(<)}(\mathbf{p_1}, \omega_1) + G^{<(>)}(\mathbf{p} + \mathbf{p_1}, \omega + \omega_1) G_{(ST)}^{>(<)}(\mathbf{p_1}, \omega_1; \mathbf{p_1} - \mathbf{q}, \omega_1 - \Omega) \Big)$$
(22)

and

$$\widetilde{\Pi}_{(ST)}^{r(a)}(\mathbf{q} + \mathbf{p}, \omega + \Omega; p, \omega) = -iTr \int \frac{d^3p_1 d\omega_1}{(2\pi)^4} \Big($$

$$G_{(ST)}^{r(a)}(\mathbf{q} + \mathbf{p} + \mathbf{p_1}, \omega + \Omega + \omega_1; \mathbf{p} + \mathbf{p_1}, \omega + \omega_1)G^{<(>)}(\mathbf{p_1}, \omega_1)$$

$$+G^{r(a)}(\mathbf{p} + \mathbf{p_1}, \omega + \omega_1)G_{(ST)}^{<(>)}(\mathbf{p_1}, \omega_1; \mathbf{p_1} - \mathbf{q}, \omega_1 - \Omega)$$

$$+G_{(ST)}^{<(>)}(\mathbf{q} + \mathbf{p} + \mathbf{p_1}, \omega + \Omega + \omega_1; \mathbf{p} + \mathbf{p_1}, \omega + \omega_1)G^{r(a)}(\mathbf{p_1}, \omega_1)$$

$$+G^{<(>)}(\mathbf{p} + \mathbf{p_1}, \omega + \omega_1)G_{(ST)}^{r(a)}(\mathbf{p_1}, \omega_1; \mathbf{p_1} - \mathbf{q}, \omega_1 - \Omega)) .$$
(23)

For a scalar residual interaction the contributions of $\widetilde{\Pi}_{(ST)}$ are nonzero only for the density response, i.e. S = T = 0.

IV. RESULTS AND DISCUSSION

The numerical solution of the equations for the in-medium vertex is exorbitantly difficult [7, 27, 28, 29]. This is mainly due to the complex structure of the spectral functions. Therefore in the following we present results only at finite temperature and for a relatively large value of the momentum transfer q. In this case the spectral functions are relatively smooth and therefore easier to handle in numerical calculations. Eqs. (17), (18), (20)-(23) are solved by iteration for each given q and Ω , using the Green's functions G dressed by the self-energy (3). Using Eq.(16) we can then calculate the response function $\Pi^r_{(ST) res}$ which accounts for the effects of the residual interaction.

In Fig. 8 we show the results for this polarization function with vertex corrections Π_{res} for q=220 MeV. The results for the imaginary part of the response function originating from the naive one-loop polarization (11) calculated with dressed propagators are represented by the dashed line. As compared to the Hartree-Fock response function (dashed-dotted line) this one-loop calculation with dressed propagators yields a significant tail at large excitation energies. As the dressed propagators include effects of two-particle one-hole and two-hole one-particle contributions to the propagation of particle and holes, the dressed response functions accounts for admixtures of two-particle two-hole contributions to the response functions. Therefore one may interprete this high-energy tail to describe a shift of the excitation strength to higher energies due to the admixture of these two-particle two-hole contributions.

If, however, we also account for the vertex corrections which are due to the residual interaction, we obtain the response functions represented by the solid line in the case of the density response and the dotted line in the case of the response for the other spin-

isospin channels. One can see that in all cases the high-energy tail obtained in the simple one-loop result is compensated by the vertex corrections. This means that the induced interaction term, which for our scalar residual interaction is present in all spin-isospin channels, is responsible for this cancellation at high energies. The difference between the density response $\Pi_{(00) \ res}$ and the response in other spin-isospin channels is due to the sub-leading vertex corrections represented by the second and third graph in Fig. 5 for K_{res} . These vertex corrections, which are specific for the density response, lead to an enhancement of the imaginary part of $\Pi_{(00) \ res}$ at small energies, which makes the final result look rather similar to the free response function without any corrections of propagator and vertex due to the residual interaction.

The free response and the consistently calculated response functions in the correlated system should fulfill several sum rules. For the scalar residual interaction the ω -sum rule takes the simple form

$$-\int_{-\infty}^{\infty} \frac{\omega d\omega}{2\pi} \operatorname{Im}\Pi_{(ST)}^{r}(\mathbf{q},\omega) = \rho \frac{\mathbf{q}^{2}}{2m} , \qquad (24)$$

in all the spin isospin channels (ST). The self-energy takes into account also the meanfield interaction, so the sum rule is only approximate and since the response Π_{res} does not include the RPA corrections on the right hand side of (24) we substitute the free mass with the effective mass. Such a modified sum rule is fulfilled to within a few percent by the response functions including vertex corrections $\Pi_{(ST)\ res}$, it is severely violated by the naive one-loop response function (11). The mean-field interaction contains spin and isospin dependent terms, and the sum rule including the mean-field part of the interaction could include a possible RPA enhancement factor [45] besides the free Fermi gas sum rule (24). When restricting the RPA response to the Landau parameter form (19) the ω -sum rule has the same form as in the Free Fermi gas (24) but with the corresponding effective mass instead of the free nucleon mass. We find that the vertex corrections due to the first diagram for the kernel K_{res} (Fig. 7), the so-called induced interaction terms, are the most important ones to bring the response function close to the one for the Free Fermi case and restore the ω -sum rule.

Adding the Hartree-Fock terms in the self-energy modifies the kernel of the equation for the dressed vertex. As explained above we take the RPA sum into account by means of the Landau parameter for the mean-field part of the interaction. The resulting response

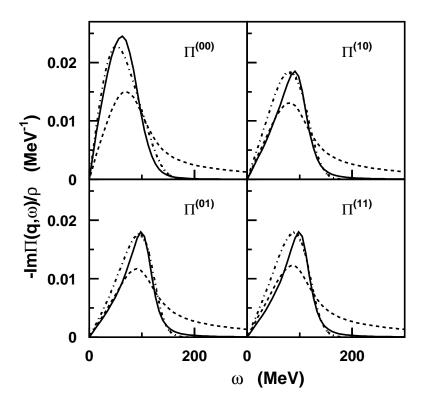


FIG. 9: The imaginary part of the response functions in different channels as function of energy for the momentum $q=220\,$ MeV, at normal nuclear density and temperature 15 MeV. The dashed-dotted line is the response function for the Gogny interaction in the Landau parameter approximation. The solid line denotes the response function including the dressing of the propagators and the vertex corrections. The dashed line is the naive one-loop response function with dressed propagators. The response function for the system with residual interaction include the modification of the response due to the modified Gogny mean-field, taken in Landau parameter approximation.

functions in different channels are shown in Fig. 9. We plot also the response function for a Fermi liquid, where the Landau parameters and the effective mass are given by the original Gogny interaction. For the density response the result is very close to the response of a Fermi liquid. In all the channels the naive one-loop polarization with dressed propagators gives a incorrect description, with long tail at large energies. In fact the Lindhard function, i.e. the one-loop polarization with HF propagators, gives a much better description of the response function [1], similar to the one including full dressed

propagators and vertices. In the spin isospin response some difference to the response of a Fermi liquid is observed, which could already be seen in Fig 8. However the constraint of the ω -sum rule makes the response in the correlated systems to lie close to the Fermi liquid one also in the nonzero spin and/or isospin channels; the overall shape of Im $\Pi_{(ST)}$ is similar to the RPA response function.

A. Neutron matter

The description of weak processes in dense nuclear matter is very important for modeling supernovae explosions and the cooling of neutron stars. In a hot and dense medium neutrinos have a short mean free path and they are effectively trapped inside the protoneutron star. The calculation of the mean free path involves nuclear correlation effects. The relevant hadronic part of the cross section can be factorized in the form of the density and spin response in matter. In this section we present a calculation the response functions in pure neutron matter.

As for the symmetric nuclear matter, the mean-field interaction has to be modified in order to take into account additional contributions from the residual interaction. In this first exploratory work we opt for a parameterization which is different in pure neutron matter and in symmetric nuclear matter. The modifications of the mean field interaction are listed in the third colum of parameters in Table I. In this way we can reproduce the same Fermi energy and similar effective mass as given by the original Gogny interaction for a range of densities between $0.4 \rho_0$ and ρ_0 . At the same time the Landau parameters are not modified drastically from their value corresponding to the original Gogny parameters displayed in the first column of Table I.

The formulas for the density and spin response in neutron matter can be written in the same way as outlined in section III. We find that for the density response the whole kernel K_{res} displayed in Fig. 7 must be considered in the BS equation, while for the spin response only the first graph in the kernel K_{res} , the induced interaction term, is nonzero $(\tilde{\Pi}_{(S)} = 0 \text{ for } S = 1)$.

The results are very similar to what we found for the symmetric nuclear matter. When both propagator and vertex modification in the medium are taken into account the response function in the correlated system is very similar to the one obtained in the Fermi

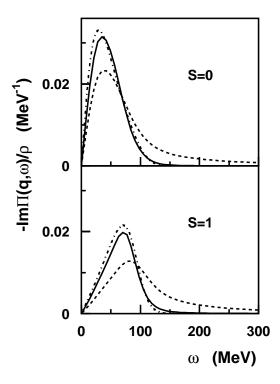


FIG. 10: The imaginary part of the response functions in different channels as function of energy for the momentum $q=220 \,\mathrm{MeV}$, in neutron matter at normal density. Symbols are the same as in Fig. 9.

liquid theory (Fig 10). It is not surprising, since the ω -sum rule has the same form as in the noninteracting system. The naive one-loop response function with dressed propagators cannot be trusted and violates the sum rule.

B. Multi-pair contributions to the response function

Both for the symmetric and pure neutron matter we find that the response function in a correlated system is very close to response function in free Fermi gas, or when the mean field is taken into account the response function is similar as in the Fermi liquid theory. This means that the cancellation of propagator dressing and vertex correction effectively drives the response of the system to the response given by the excitation of a single particle-hole pair.

However, this result is not general. This cancellation is due to the particular form of the residual interaction, which we have considered to be scalar in spin and isospin. This

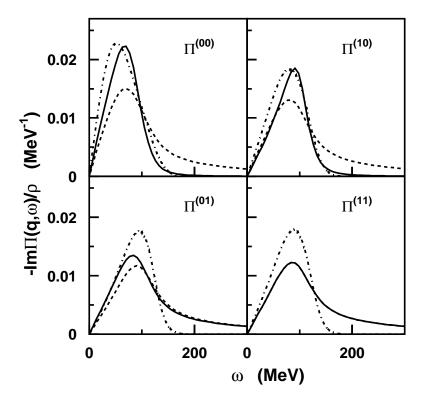


FIG. 11: The imaginary part of the response functions in different channels as function of energy and q = 220 MeV for the isospin dependent residual interaction (25). Symbols are the same as in Fig. 9. The ST = 11 response function with vertex and propagator dressing is the same as the naive one-loop result with only propagator dressing.

also leads to the simple form (24) of the ω -sum rule in all the channels.

Therefore, for the discussion in this section we modify the residual interaction and assume it to be isospin dependent in the form

$$V_{res}(\mathbf{r_1} - \mathbf{r_2})P_{\tau} = V_0 e^{-(\mathbf{r_1} - \mathbf{r_2})^2/2\eta^2} P_{\tau} . \tag{25}$$

In symmetric nuclear matter the single-particle self-energy is the same as obtained for the scalar residual interaction. Therefore the same modified Gogny parameterization of the mean field interaction is used as in section II. The kernel K_{res} of the BS equation, however, is different than in the case of a scalar residual interaction. For the ST = 00channel all the three graphs for K_{res} in Fig. 5 contribute. However, for the response function in channels with isospin T = 1 the first diagram in K_{res} , the induced interaction term does not contribute. We have found previously that this induced interaction diagram in the vertex dressing is crucial for the suppression of the high-energy tail in the response function. This led to the restoration of the ω -sum rule and made the response function similar to the one in the free Fermi gas.

So if this induced interaction contribution to the vertex dressing is absent (channels ST = 01, 11) we expect a strong modification of the response function by the residual interaction. The vertex and propagator dressing do no longer cancel. In fact, in the channel ST = 11 there are no vertex corrections at all $(K_{res} = 0)$, and the response function has the same form as a naive one-loop calculation with dressed propagators.

In Fig. 11 the response functions obtained with the isospin dependent interaction (25) are compared to the response functions from the Fermi liquid theory. For T=0 channels the correlated response function is similar to the one particle-one hole response function. On the other hand, the isovector response is closer to the naive one-loop result.

For the T=1 channels the ω -sum rule is different than in the free Fermi gas. The residual interaction in the Hamiltonian gives a modification factor in the ω - sum rule at finite momentum q

$$-\int_{-\infty}^{\infty} \frac{\omega d\omega}{2\pi} \operatorname{Im}\Pi_{(ST)\ res}^{r}(\mathbf{q},\omega) = \rho \frac{\mathbf{q}^{2}}{2m} + \frac{2}{3} \int d\mathbf{r}_{1} d\mathbf{r}_{2} V_{res}(\mathbf{r}_{1} - \mathbf{r}_{2}) \tau_{1} \tau_{2} \Psi^{\dagger}(r_{1}) \Psi^{\dagger}(r_{2}) \Psi(r_{2}) \Psi(r_{1}) \left(e^{i\mathbf{q}(\mathbf{r}_{1} - \mathbf{r}_{2})} - 1\right) (26)$$

This enhancement of the sum rule in the T=1 response is consistent with observed long tail in the response function Im Π at large energies. A nonzero value of the imaginary part of the response function at large energies is not kinematically allowed by one particle-one hole configuration with on shell propagation. Nonzero contribution do appear due to the dressing of the single-particle propagator by the self-energy from the residual interaction. Such a dressed propagation involves nucleons which are put off shell by the scattering on other particles in the medium. For the isospin dependent interaction and T=1 response these off-shell propagation effects are not canceled by vertex corrections. In the case of the residual interaction of the form (25) off-shell nucleons couple in the same way as free nucleons to isovector potentials.

For a general residual interaction containing scalar, spin, and isospin dependent terms, we expect that the spin and isospin responses in a correlated system lie in between the naive one-loop result and the Fermi liquid theory result.

C. Collective modes

The response function may show a pronounced peak at a certain excitation energy. This is a collective mode, which corresponds to the excitation of a single collective state in the interacting system. Depending on the spin-isospin character of the response these are the zero sound mode, spin or isospin waves. In nuclear physics the isovector response is of particular importance [46]. In finite nuclei it shows up as the giant dipole resonance, which has extensively been studied.

Within the Fermi Liquid theory a collective excitation at zero temperature is a discrete peak in the imaginary part of the response function. The state corresponding to the collective excitation cannot couple to the incoherent one particle-one hole excitations. At finite temperature such a coupling is possible, it can be calculated and the width of the collective state at finite temperature is usually small. The collective state can acquire a finite width (also at zero temperature) due to a coupling to multi pair configurations [1]. The description of this damping of the collective states from such admixtures goes beyond the usual Fermi liquid theory. In the preceeding subsection we have seen that a isospin dependent residual interaction can produce correlations in the response function which correspond to the admixture of multi pair configurations.

For the chosen temperature and kinematics, however, the Hamiltonian considered here does not lead to strong collective modes in any of the response functions. To study the role of the multi pair configurations on the collective modes we increase the value of the Landau parameters. In Fig. 12 we present the density response function assuming a Landau parameter $F_0 = 4$. In this case the RPA response function shows a well defined peak. The relatively high temperature yields a collective zero sound mode with a finite width due to the coupling to thermally excited one particle-one hole states. The calculation including multi-pair correlations in the system from the residual interaction does also show a collective state in the density response. The position of this collective mode is almost at the same place as for the Fermi liquid (Fig. 12).

The ST=00 response function with propagator and vertex corrections is almost the same as in the free Fermi gas. The difference is that at high energy the response function Im $\Pi_{(00)\ res}$ is slightly larger than the finite temperature Lindhard function. This causes the collective mode to have a larger width in the system with residual interactions.

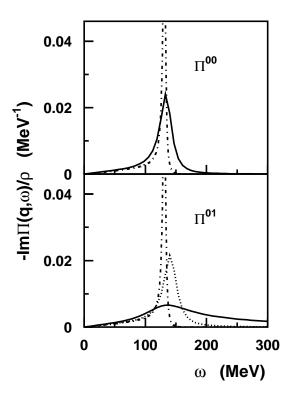


FIG. 12: The imaginary part of the response functions as function of energy and q = 220 MeV for the isospin dependent residual interaction (25) (solid lines) and for the scalar residual interaction (2) (dotted line). Dashed-dotted line are results for a Fermi liquid at finite temperature. In order to get a collective mode for the chosen momentum and temperature the Landau parameters are set by hand to F_0 , $F_0' = 4$.

The damping of the zero sound has two origins in a system interacting with a residual interaction: a finite temperature width and a width due to the coupling to multi-pair states.

In the lower panel of Fig. 12 the isovector response function is shown for the Landau parameter $F_0' = 4$. The Fermi liquid theory predicts the presence of a well defined collective state. The finite width is due of course to the finite temperature. For the scalar residual interaction the response function in the correlated system shows a collective state at similar energy. It has a larger width due to the contribution of multi-pair configurations, analogously as in the density response.

The isospin dependent residual interaction leads to a response Im $\Pi_{(01)}$ res with a long tail at large energies. Due to the large contribution of these configurations the width

of the collective states in the isovector response is very large. In fact, the collective mode disappears. The disappearance of the collective mode is an extreme case where the coupling to multi-pair state is not reduced by vertex corrections (special case of the interaction (25)).

For a general interaction we expect a whole range of behavior depending on the energy of the collective state and on the strength of isospin dependent terms in the residual interaction. The collective state would be generally broader than in the Fermi liquid theory, due to the coupling to multi-pair configurations. In some cases this coupling can lead to a disappearance of the collective state. The same phenomena are expected also for the spin wave collective state in the presence of spin dependent residual interactions.

V. CONCLUSIONS

The aim of this paper has been a consistent study of correlation effects on the response function going beyond the usual HF plus RPA approach. For that purpose we consider a mean-field interaction and a residual interaction. This residual interaction generates contributions to the self-energy of the nucleons, which describe the admixture of two-hole one-particle and two-particle one-hole configurations to the single-particle propagator.

The response function using these dressed propagators in a one-loop approximation will in general violate the energy weighted sum rule for the excitation function. These sum-rules are fulfilled only if the response functions are calculated employing a consistent treatment of propagator and vertex corrections following the recipe of Baym and Kadanoff [25, 26]. A scheme for such a consistent treatment of correlation effects in the response function of nuclear matter is outlined and numerical results are presented for symmetric nuclear matter and pure neutron matter at finite temperature.

Assuming a residual interaction of scalar-isoscalar form it turns out that the effects originating from propagator corrections are to a large extent compensated by vertex corrections in the response function for all spin-isospin channels. The induced interaction terms in particular are responsible for the compensation of the correlation effects in the single-particle propagator.

If, however, a residual interaction with non-trivial spin isospin structure is considered this cancellation of correlation effects is removed in specific spin-isospin channels.

Consequences for the damping of collective excitation modes due to these admixtures of multi-particle multi-hole contributions are discussed.

The present investigation employs residual interaction with a rather simple spin-isospin structure. More realistic interaction models should be investigated in extended kinematical regions of q and Ω to obtain detailed information on the importance of correlation effects on the nuclear response for the various excitation modes.

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